**ASSIGNMENTS 11 SOLUTION**

**1. Given X be a discrete random variable with the following PMF**

**1. Find the range RX of the random variable X.**

**2. Find P(X ≤ 0.5)**

**3. Find P(0.25&lt;X&lt;0.75)**

**4. P(X = 0.2|X&lt;0.6)**

**2. Two equal and fair dice are rolled, and we observed two numbers X and Y.**

**1. Find RX, RY, and the PMFs of X and Y.**

**2. Find P(X = 2,Y = 6).**

**3. Find P(X&gt;3|Y = 2).**

**4. If Z = X + Y. Find the range and PMF of Z.**

**5. Find P(X = 4|Z = 8).**

**3. In an exam, there were 20 multiple-choice questions. Each question had 44 possible**

**options. A student knew the answer to 10 questions, but the other 10 questions were unknown**

**to him, and he chose answers randomly. If the student X&#39;s score is equal to the total number**

**of correct answers, then find out the PMF of X. What is P(X&gt;15)?**

**4. The number of students arriving at a college between a time interval is a Poisson random**

**variable. On average, 10 students arrive per hour. Let Y be the number of students arriving**

**from 10 am to 11:30 am. What is P(10&lt;Y≤15)?**

**5.Two independent random variables, X and Y,are given such that X~Poisson(α) and**

**Y~Poisson(β). State a new random variable as Z = X + Y. Find out the PMF of Z.**

**6. There is a discrete random variable X with the pmf.**

**If we define a new random variable Y = (X + 1)2 then**

**1. Find the range of Y.**

**2. Find the pmf of Y.**

**2.Assuming X is a continuous random variable with PDF**

**1. Find EX and Var(X).**

**2. Find P(X ≥ ).**

**2. If X is a continuous random variable with pdf**

**3. If X~Uniform and Y = sin(X), then find f Y (y).**

**4. If X is a random variable with CDF**

**1. What kind of random variable is X: discrete, continuous, or mixed?**

**2. Find the PDF of X, fX(x).**

**3. Find E(e X ).**

**4. Find P(X = 0|X≤0.5).**

**2. There are two random variables X and Y with joint PMF given in Table below**

**1. Find P(X≤2, Y≤4).**

**2. Find the marginal PMFs of X and Y.**

**3. Find P(Y = 2|X = 1).**

**4. Are X and Y independent?**

**SOLUTIONS**

1. Since two fair dice are rolled, X and Y can take values from 1 to 6 with equal probability. Therefore, RX = RY = {1, 2, 3, 4, 5, 6}. The PMF of X and Y is given by: P(X = k) = P(Y = k) = 1/6 for k = 1, 2, 3, 4, 5, 6
2. P(X = 2,Y = 6) = P(X = 2) \* P(Y = 6) = 1/6 \* 1/6 = 1/36
3. P(X>3|Y=2) = P(X>3∩Y=2)/P(Y=2) = P(X>3∩Y=2)/P(Y=2∩X≤6) //Since X cannot be more than 6 = P(X>3∩Y=2)/P(Y=2) = P(X>3) \* P(Y=2) / P(Y=2) = P(X>3) = 3/6 = 1/2
4. The range of Z is the set of all possible values that Z can take. Since Z = X + Y, the range of Z is the set of all possible sums of values of X and Y. If X and Y are both discrete random variables, then Z is also a discrete random variable, and we can find its probability mass function (PMF) as follows:

PMF of Z(z) = P(Z = z) = P(X + Y = z)

We can compute the PMF of Z by considering all possible combinations of values of X and Y that add up to z:

PMF of Z(z) = Σ P(X = x, Y = z - x)

where the sum is taken over all possible values of x such that x + (z - x) = z.

1. Using Bayes' theorem, we have:

P(X = 4|Z = 8) = P(X = 4, Y = 4|Z = 8) / P(Z = 8)

Since Z = X + Y, we can rewrite the numerator as:

P(X = 4, Y = 4|Z = 8) = P(X = 4, Y = 4, X + Y = 8)

Since X and Y are independent, we have:

P(X = 4, Y = 4, X + Y = 8) = P(X = 4) \* P(Y = 4) = e^(-λ) \* λ^4 / 4! \* e^(-λ) \* λ^4 / 4!

where λ = α + β. Thus,

P(X = 4|Z = 8) = (e^(-λ) \* λ^4 / 4! \* e^(-λ) \* λ^4 / 4!) / P(Z = 8)

where P(Z = 8) is given by the PMF of Z that we computed in part (4).

1. Let X be the number of correct answers. Since the student knew the answer to 10 questions, X has a binomial distribution with parameters n=20 and p=10/44, where n is the number of trials (number of questions) and p is the probability of success (getting the correct answer).

The PMF of X is given by:

P(X=k) = (n choose k) \* p^k \* (1-p)^(n-k)

where (n choose k) is the binomial coefficient which represents the number of ways to choose k correct answers out of n questions.

For k = 0 to 10, P(X=k) = 0 because the student knew the answer to 10 questions.

For k = 11 to 20, P(X=k) = (20 choose k) \* (10/44)^k \* (34/44)^(20-k)

To find P(X>15), we can sum the probabilities for k = 16 to 20:

P(X>15) = P(X=16) + P(X=17) + P(X=18) + P(X=19) + P(X=20) = (20 choose 16) \* (10/44)^16 \* (34/44)^4 + (20 choose 17) \* (10/44)^17 \* (34/44)^3 + (20 choose 18) \* (10/44)^18